Aggregate Production Planning

Manufacturing Systems Overview

- Product Design
- Process Planning
- Production System Design
- Production Planning
- Operational Planning
- Shop Floor Control
- Execution

Aggregate Production Planning

- Competing Objectives:
  - Chase
    - Reacting quickly to changes in demand
  - Stability
    - Retaining stability in workforce and production

Aggregate Planning

- Definition:
  - How many employees a firm should retain, and, for a manufacturing firm, the quantity and mix of products to be produced over some planning horizon.
  - Sometimes referred to as macro production planning.
- An aggregate planning methodology is designed to translate demand forecasts into a blueprint for planning staffing and production levels for the firm over a predetermined planning horizon.

Aggregate Units of Production

- If the types of parts are similar, an aggregate unit of production corresponds to an “average” item.
- Otherwise, it would correspond to some other unit of measure, such as:
  - weight
  - volume
  - work required
  - dollar value

Production Planning Hierarchy

Aggregate Planning:

- Forecast of aggregate demand for t-period planning horizon
- Determination of aggregate production and workforce levels for t-period planning horizon
- Production levels by item by time period
- Detailed timetable for production and assembly of components and subassemblies

Aggregate Planning:

- Competing Objectives:
  - Chase
  - Reacting quickly to changes in demand
  - Stability
  - Retaining stability in workforce and production
Aggregate Units of Production

- Example

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Worker hours required to produce</th>
<th>Selling price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A5532</td>
<td>4.2</td>
<td>$ 285</td>
</tr>
<tr>
<td>K4242</td>
<td>4.9</td>
<td>$ 345</td>
</tr>
<tr>
<td>L9898</td>
<td>5.1</td>
<td>$ 395</td>
</tr>
<tr>
<td>L3800</td>
<td>5.2</td>
<td>$ 425</td>
</tr>
<tr>
<td>M2624</td>
<td>5.4</td>
<td>$ 525</td>
</tr>
<tr>
<td>M3880</td>
<td>5.8</td>
<td>$ 725</td>
</tr>
</tbody>
</table>

Aggregate Production Planning

- Goal of aggregate planning
  - To determine aggregate production quantities and the levels of resources required to achieve these production goals.

- Issues involved with aggregate planning
  - Smoothing
  - Bottleneck problems
  - Planning horizon
  - Treatment of demand

Example

<table>
<thead>
<tr>
<th>Month</th>
<th>Net Predicted Demand</th>
<th>Net Cumulative Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>January*</td>
<td>1280-500</td>
<td>780</td>
</tr>
<tr>
<td>February</td>
<td>640</td>
<td>1,420</td>
</tr>
<tr>
<td>March</td>
<td>900</td>
<td>2,320</td>
</tr>
<tr>
<td>April</td>
<td>1,200</td>
<td>3,520</td>
</tr>
<tr>
<td>May</td>
<td>2,000</td>
<td>5,520</td>
</tr>
<tr>
<td>June**</td>
<td>1,400+600</td>
<td>7,520</td>
</tr>
</tbody>
</table>

* Forecasted demand - stock on hand
** Forecasted demand + ending stock requirements

Costs in Aggregate Planning

- Smoothing Costs
- Holding Costs
- Shortage (Backorder) Costs
- Regular Time Costs
- Overtime/Subcontracting Costs

Example (cont.)
Example (cont.)

Currently 300 workers employed at the plant.

Relevant Costs:

- Cost of hiring one worker: $500
- Cost of firing one worker: $1,000
- Cost of holding one unit of inventory for one month: $80

Example - Zero Inventory Strategy

<table>
<thead>
<tr>
<th>Month</th>
<th>Days</th>
<th>Units per Worker</th>
<th>Forecast demand</th>
<th>Min Workers Required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>20</td>
<td>2.931</td>
<td>760</td>
<td>267</td>
</tr>
<tr>
<td>Feb</td>
<td>24</td>
<td>3.517</td>
<td>640</td>
<td>182</td>
</tr>
<tr>
<td>Mar</td>
<td>18</td>
<td>2.638</td>
<td>900</td>
<td>342</td>
</tr>
<tr>
<td>Apr</td>
<td>26</td>
<td>3.810</td>
<td>1200</td>
<td>315</td>
</tr>
<tr>
<td>May</td>
<td>22</td>
<td>3.224</td>
<td>2000</td>
<td>621</td>
</tr>
<tr>
<td>Jun</td>
<td>15</td>
<td>2.198</td>
<td>2000</td>
<td>910</td>
</tr>
</tbody>
</table>

Zero Strategy Cost:
- Hiring: $377,500
- Firing: $145,000
- Holding: $49,920
- Total: $572,420

Example - Stable Workforce Strategy

<table>
<thead>
<tr>
<th>Month</th>
<th>Days</th>
<th>Forecast demand</th>
<th>Cumulative Units per Worker</th>
<th>Workers required</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>20</td>
<td>780</td>
<td>85</td>
<td>1432</td>
</tr>
<tr>
<td>Feb</td>
<td>24</td>
<td>640</td>
<td>252</td>
<td>1360</td>
</tr>
<tr>
<td>Mar</td>
<td>18</td>
<td>900</td>
<td>602</td>
<td>1920</td>
</tr>
<tr>
<td>Apr</td>
<td>26</td>
<td>1200</td>
<td>1170</td>
<td>2020</td>
</tr>
<tr>
<td>May</td>
<td>22</td>
<td>2000</td>
<td>1750</td>
<td>343</td>
</tr>
<tr>
<td>Jun</td>
<td>15</td>
<td>2000</td>
<td>1500</td>
<td>411</td>
</tr>
</tbody>
</table>

Stable Workforce Strategy Cost:
- Hiring: $55,500
- Firing: $0
- Holding: $524,480
- Total: $579,980

Example - Stable Workforce Strategy

<table>
<thead>
<tr>
<th>Month</th>
<th>Days</th>
<th>Forecast demand</th>
<th>Cumulative Units per Worker</th>
<th>Ending Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>20</td>
<td>2.931</td>
<td>1204</td>
<td>424</td>
</tr>
<tr>
<td>Feb</td>
<td>24</td>
<td>3.517</td>
<td>1445</td>
<td>829</td>
</tr>
<tr>
<td>Mar</td>
<td>18</td>
<td>2.638</td>
<td>1094</td>
<td>118</td>
</tr>
<tr>
<td>Apr</td>
<td>26</td>
<td>3.810</td>
<td>1156</td>
<td>1179</td>
</tr>
<tr>
<td>May</td>
<td>22</td>
<td>3.224</td>
<td>1325</td>
<td>6824</td>
</tr>
<tr>
<td>Jun</td>
<td>15</td>
<td>2.198</td>
<td>953</td>
<td>7527</td>
</tr>
</tbody>
</table>

Total Cost:
- Hiring: $55,500
- Firing: $0
- Holding: $524,480
- Total: $579,980

“Optimal” Strategy

- Objective: Minimize (Hiring/Firing Cost + Inventory Holding Cost)
- Constraints:
  - Workforce Constraints
  - Material Conservation Constraints
  - Capacity/Consistency Constraints
  - Production level must match workforce levels
Example - “Optimal” Strategy

- Total Cost = $379,320.90
- Solution
  - Fire 27 people in period 1
  - Hire 465 people in period 5

<table>
<thead>
<tr>
<th>Month</th>
<th>Production</th>
<th>Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>800</td>
<td>20</td>
</tr>
<tr>
<td>Feb.</td>
<td>960</td>
<td>340</td>
</tr>
<tr>
<td>Mar.</td>
<td>720</td>
<td>160</td>
</tr>
<tr>
<td>Apr.</td>
<td>1040</td>
<td>0</td>
</tr>
<tr>
<td>May</td>
<td>2378</td>
<td>378</td>
</tr>
<tr>
<td>Jun.</td>
<td>1622</td>
<td>600</td>
</tr>
</tbody>
</table>

Constant Production Strategies

Varying Production Levels

Aggregate Planning Formulations

- Notation
  - $a_{ij}$ = time required on workstation $j$ to produce one unit of product $i$
  - $c_j$ = capacity of workstation $j$ in period $t$ (units consistent with $a_{ij}$)
  - $r_i$ = net profit from one unit of product $i$
  - $h_i$ = cost to hold one unit of product $i$ for one period
  - $X_{it}$ = amount of product $i$ produced in period $t$
  - $S_{it}$ = amount of product $i$ sold in period $t$
  - $I_{it}$ = inventory of product $i$ at end of period $t$

Product Mix Planning

Linear Programming Formulation

\[
\begin{align*}
\text{max} & \sum_{i,j} c_{ij} S_{ij} - a_{ij} I_{ij} \\
\text{s.t. } & d_j \leq S_j \leq d_j + \sum_{i,j} a_{ij} X_{ij} \\
& \sum_{i,j} a_{ij} X_{ij} \leq c_j \\
& I_{ij} = I_{ij-1} + X_{ij} - S_{ij} \\
& X_{ij}, S_{ij}, I_{ij} \geq 0
\end{align*}
\]

Extensions

- Other resource constraints
- Utilization considerations
  - $q \leq 100\%$
- Backorders
  - $I_{ij} = I_{ij-1} - I_{ij}$
- Overtime
  - Supplement regular, available capacity at a cost
- Yield loss
  - Workstations early in the line must process more material to account for fallout later in the line
  - Effective capacity is reduced
**Notation**

- \( b \) = number of person-hours required to produce one unit of product
- \( l(t) \) = cost of regular time (overtime)
- \( e(t) \) = cost to increase (decrease) workforce
- \( W_t \) = workforce in period \( t \)
- \( H_t \) = increases (decreases) in workforce in period \( t \)
- \( O_t \) = overtime in period \( t \)

**Workforce Planning**

**Linear Programming Formulation**

\[
\begin{align*}
\text{max} & \quad \sum_{t=1}^{T} \left[ (S_t - b(t) - l(t) - e(t) - O_t - e(t)) X_t \right] \\
\text{st:} & \quad 0 \leq S_t \leq T_t \\
& \quad a_t X_t \leq e_t \\
& \quad I_t = I_{t-1} + X_t - S_t \\
& \quad W_t = W_{t-1} + H_t - F_t \\
& \quad bX_t \leq W_t + O_t \\
& \quad X_t, S_t, I_t, O_t, W_t, H_t, F_t \geq 0 
\end{align*}
\]

**Production Planning Hierarchy**

- **Forecast of aggregate demand for period**
- **Aggregate Production Plan**
  - Determination of aggregate production and work force levels for period planning horizon
- **Master Production Schedule**
  - Production levels by item by time period
- **Materials Requirement Plan**
  - Detailed timetable for production and assembly of components and subassemblies

**Detailed Production Planning**

- After aggregate plans are developed they must be disaggregated into specific products per period
  - This may not be straightforward depending on the method and level of aggregation
- Production plan is for end items - independent demand
- Most products are assemblies and consist of many component parts - some made, some bought
- Production plan must be “exploded” according to the bill of materials to generate plans for component items - dependent demand
  - MRP is often used for this step

**Detailed Production Planning (cont.)**

- Production (and purchasing) plans for the dependent demand must be determined.
- Two common strategies
  - Lot-for-lot
  - Batching
    - How to determine batch (lot) size?
      - Depends on setup/ordering cost, inventory holding cost, lead time uncertainty, etc.
- A production plan for each component part and assembled product by period is obtained. These production requirements are used to drive the operational planning models.